Composable Core-sets for Diversity and Coverage Maximization

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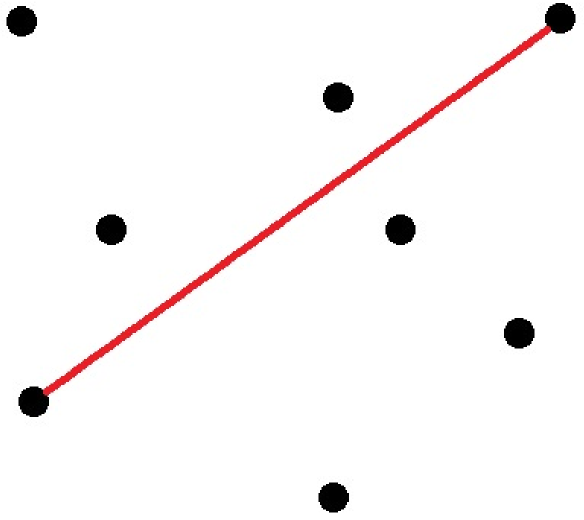
Core-Set Definition

- **Setup**
  - Set of n points $\boldsymbol{P}$ in d-dimensional space
  - Optimize a function f



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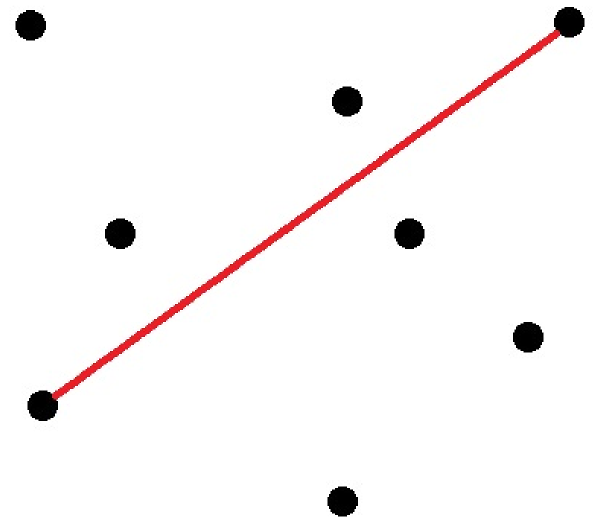
- Maximization: $\frac{f_{opt}(P)}{c} \leq f_{opt}(S) \leq f_{opt}(P)$



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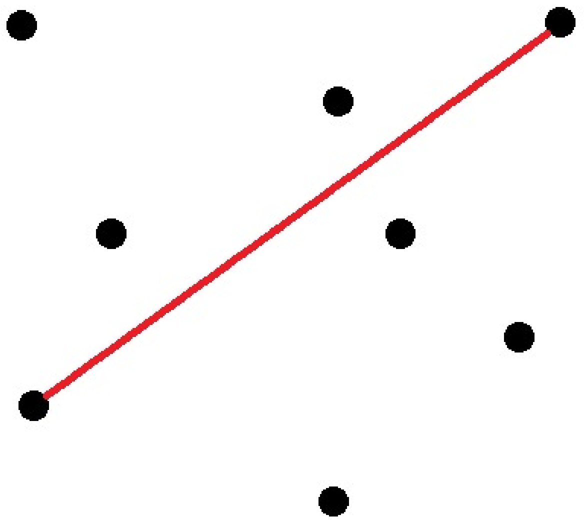
  - Optimization Function: Distance of the two farthest points



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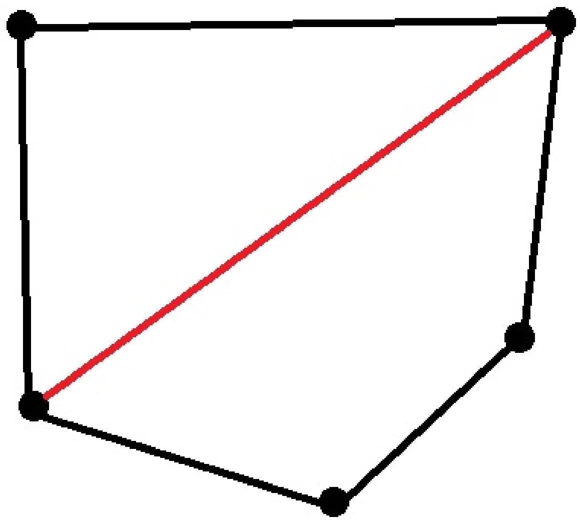
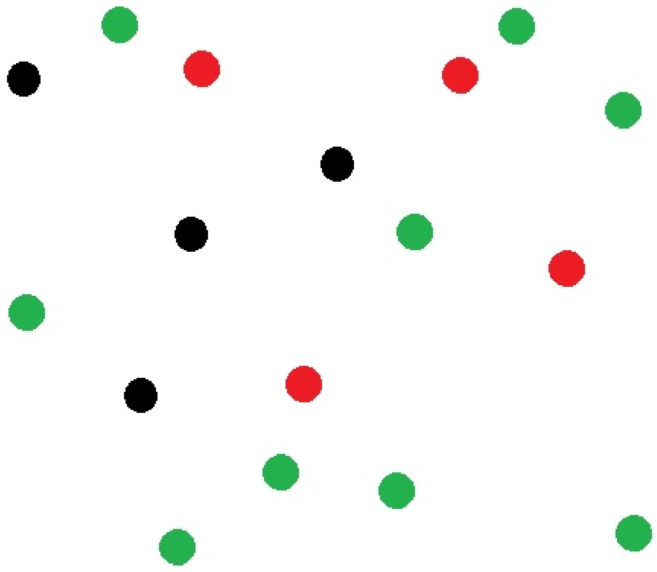
  - Optimization Function: Distance of the two farthest points
  - 1-Core-set: Points on the convex hull.



Composable Core-sets

- **Setup**
  - $\boldsymbol{P_1, P_2, \ldots, P_m}$ are set of points in d-dimensional space
  - Optimize a function f over their union $\boldsymbol{P}$.



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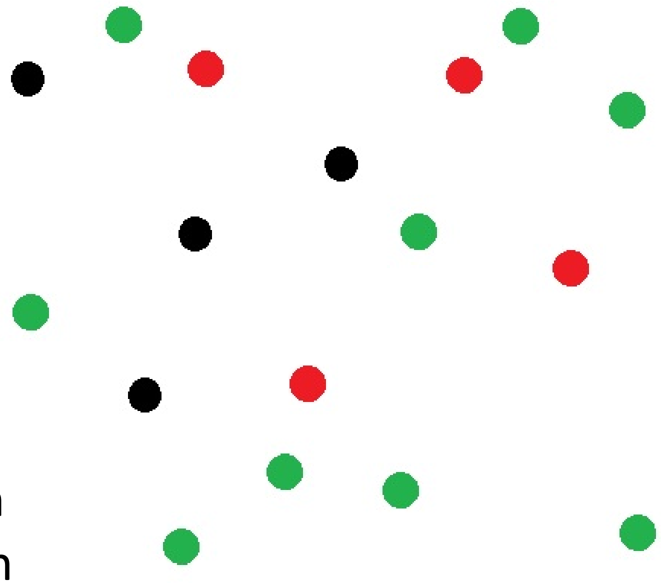

- **c-Composable Core-sets:** Subsets of points $S_1 \subset P_1, S_2 \subset P_2, \ldots, S_m \subset P_m$ points such that the solution of the union of the core-sets approximates the solution of the point sets.
  - Maximization :

$$\frac{1}{c} f_{opt}(P_1 \cup \cdots \cup P_m) \leq f_{\mathrm{opt}}(S_1 \cup \cdots \cup S_m) \leq f_{opt}(P_1 \cup \cdots \cup P_m)$$

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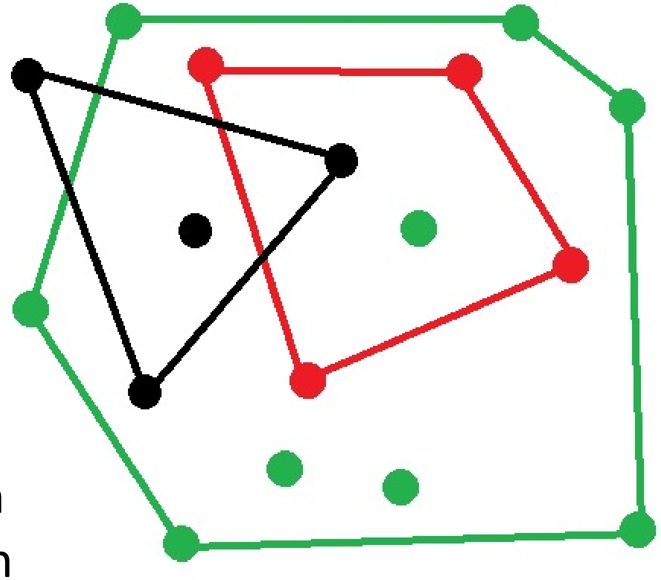

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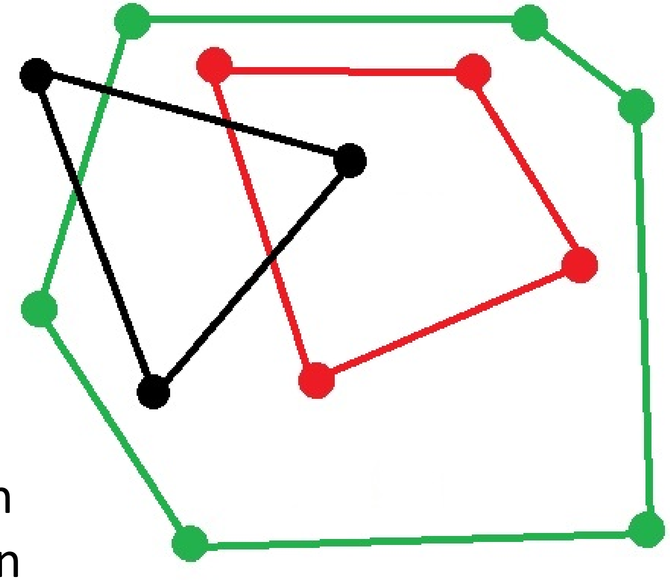

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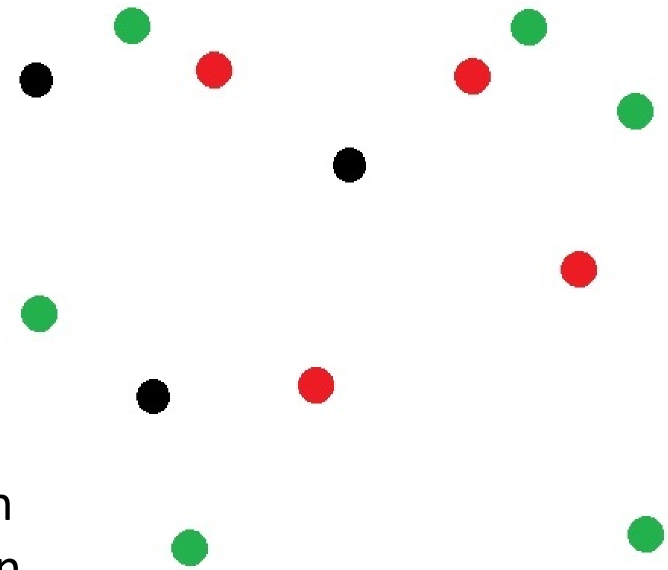

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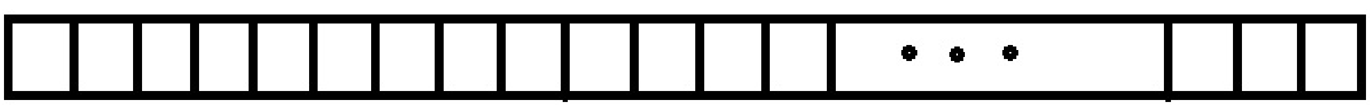

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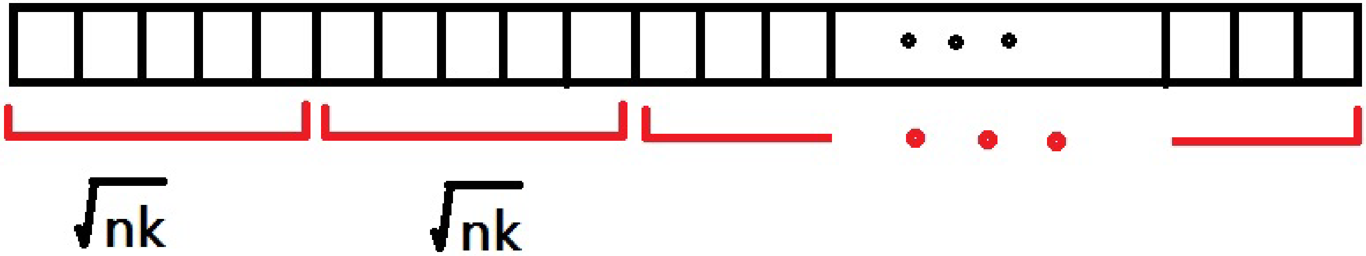
  - Chunks of size $\sqrt{nk}$, thus number of chunks = $\sqrt{n/k}$



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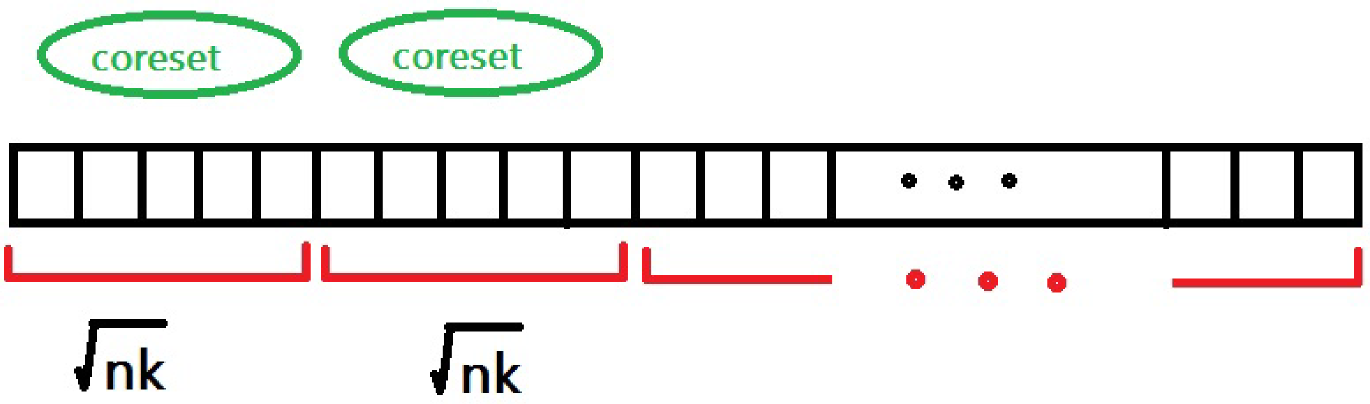
- **c-Composable Core-set of size k**
  - Chunks of size $\sqrt{nk}$, thus number of chunks = $\sqrt{n/k}$
  - Core-set for each chunk
  - Total Space: $k\sqrt{n/k} + \sqrt{nk} = O(\sqrt{nk})$
  - Approximation Factor: c



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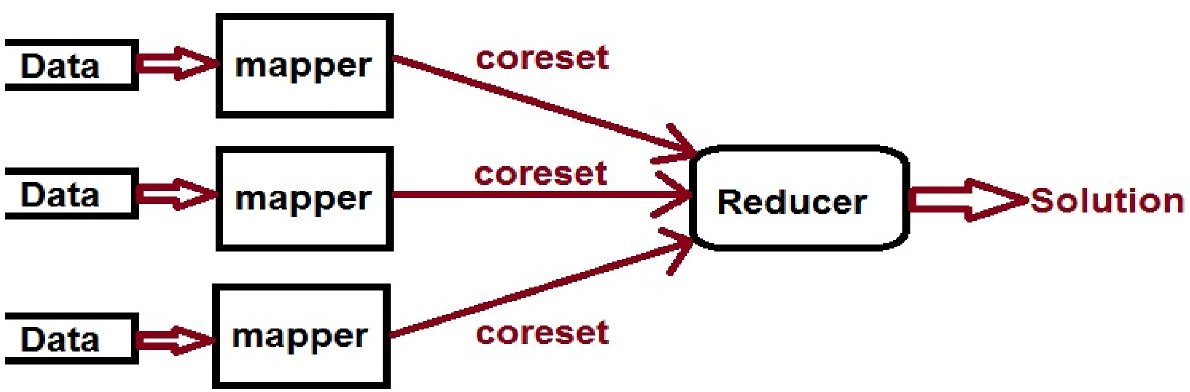
- **Map-Reduce Model:**
  - One round of Map-Reduce
  - $\sqrt{n/k}$ mappers each getting $\sqrt{nk}$ points
  - Mapper computes a composable core-set of size k
  - Will be passed to a single reducer



Applications – Similarity Search

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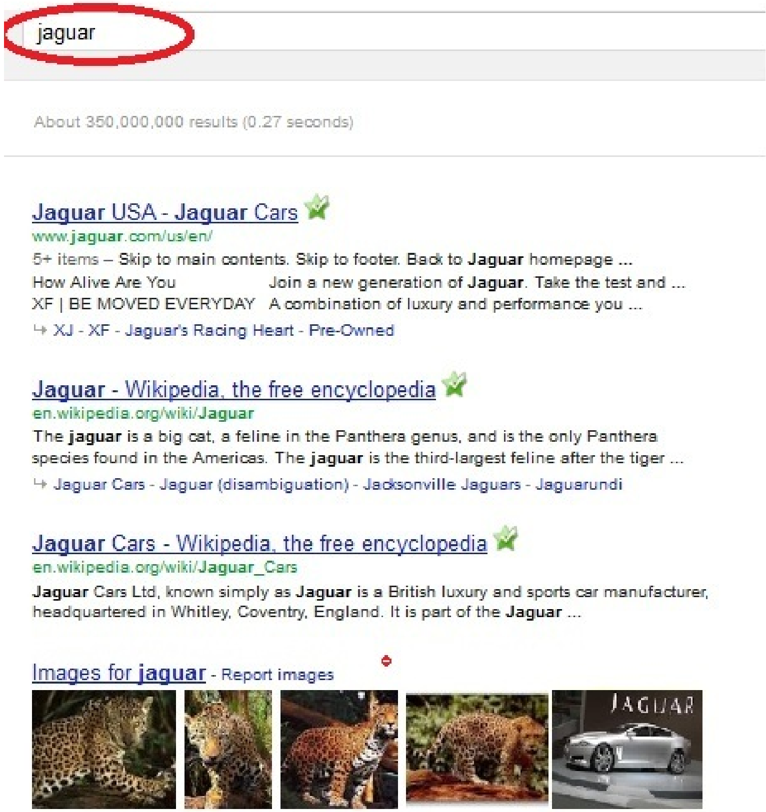

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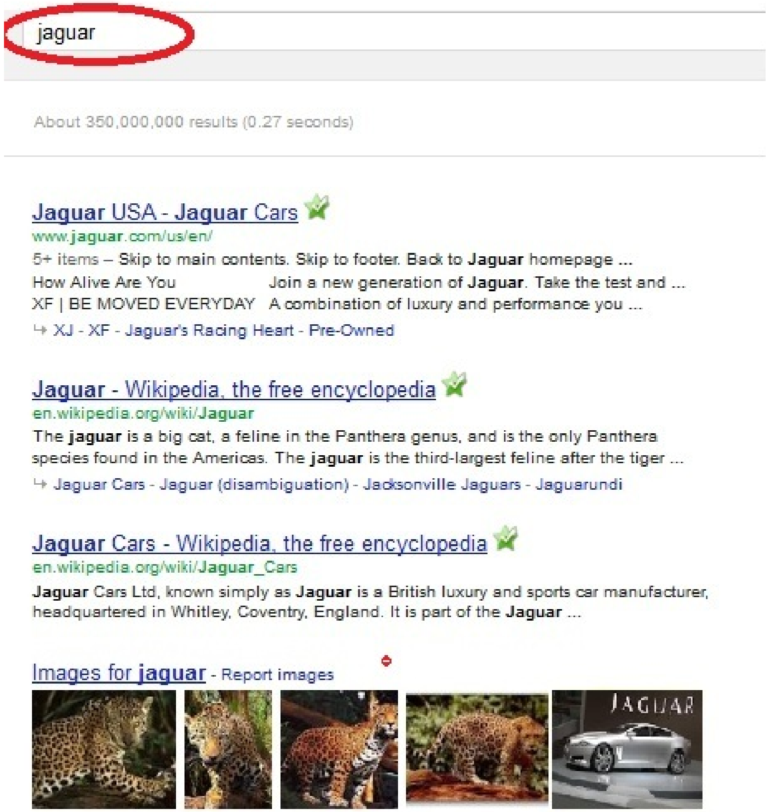

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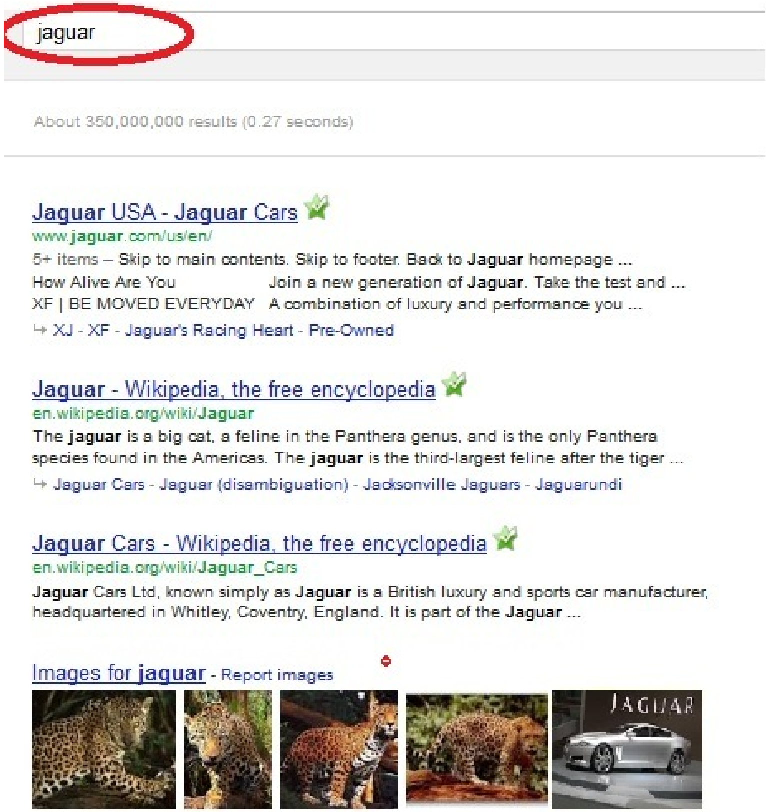
  - Diverse Near Neighbor Problem [Abbar, Amer-Yahia, Indyk, Mahabadi WWW'13] [Abbar, Amer-Yahia, Indyk, Mahabadi, Varadarajan, SoCG'13]



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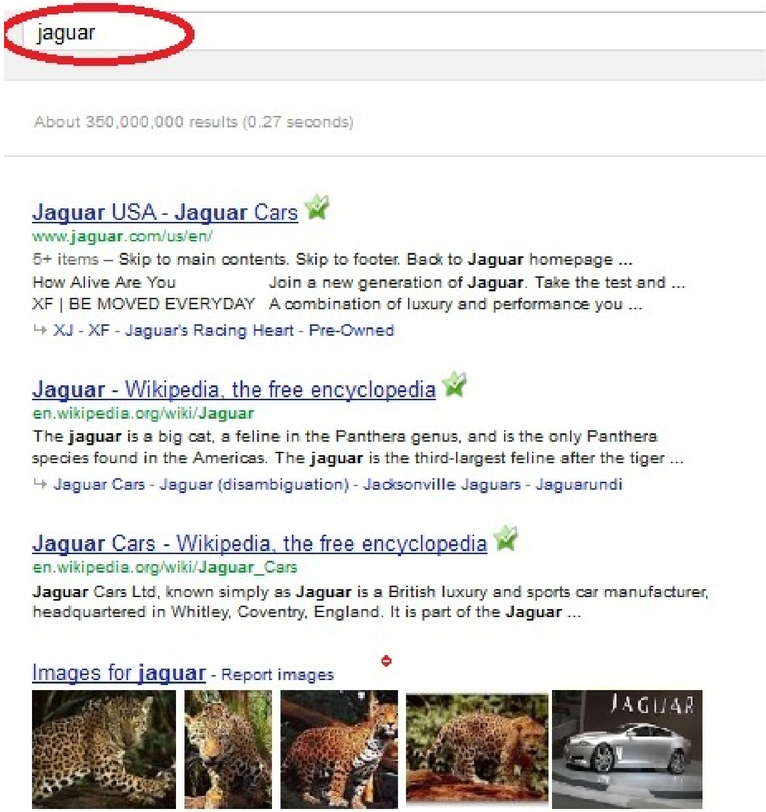
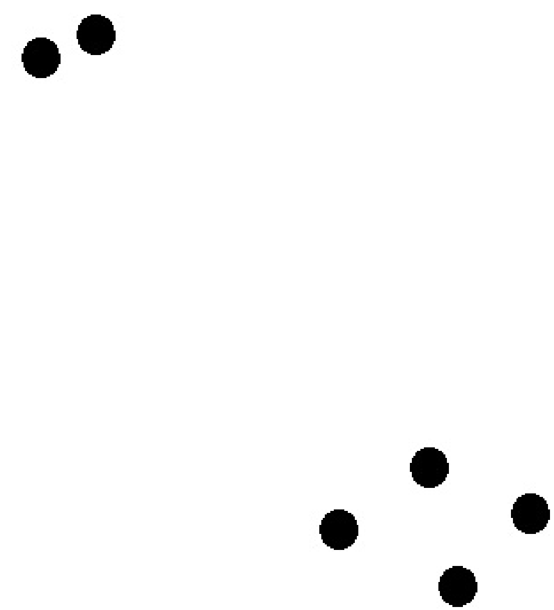
    - uses Locality Sensitive Hashing (LSH) and Composable Core-sets techniques.



Diversity Maximization Problem

- A set of n points P in metric space $(\Delta, dist)$
- Optimization Problem:
  - Find a subset of k points S which maximizes Diversity



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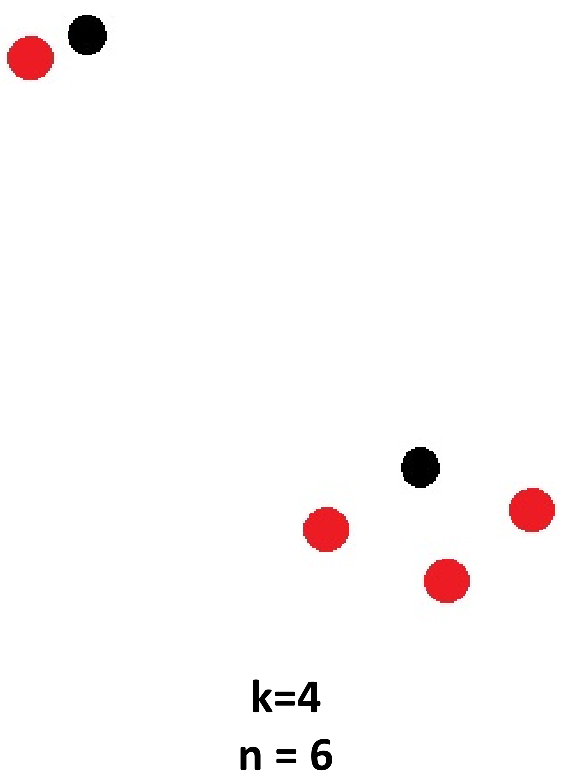
  - Minimum pairwise distance (Remote Edge)



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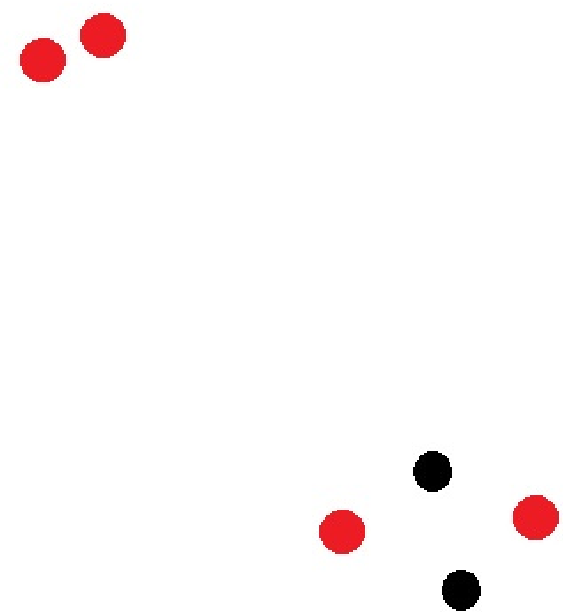

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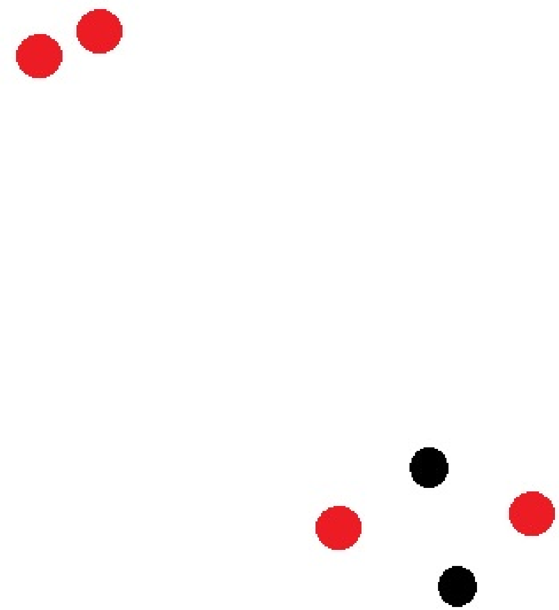
- Long list of variants [Chandra and Halldorsson '01]



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Diversity Functions

Diversity function over a set S of k point	Description
Remote-edge	Minimum Pairwise Distance: $\min_{\{p,q \in S\}} dist(p,q)$
Remote-clique	Sum of Pairwise Distances : $\sum_{\{p,q \in S\}} dist(p,q)$
Remote-tree	Weight of Minimum Spanning Tree (MST) of the set S
Remote-cycle	Weight of minimum Traveling Salesman Tour (TSP) of the set S
Remote-star	Weight of minimum star: $\min_{\{p \in S\}} \sum_{\{q \in S\}} dist(p,q)$
Remote-Pseudoforest	Sum of the distance of each point to its nearest neighbor $\sum_{\{p \in S\}} \min_{\{q \in S\}} dist(p,q)$
Remote-Matching	Weight of minimum perfect Matching of the set S
Max-Coverage	How well the points cover each coordinate $\sum_{i=1}^{d} \max_{p \in S} p_i$

Our Results

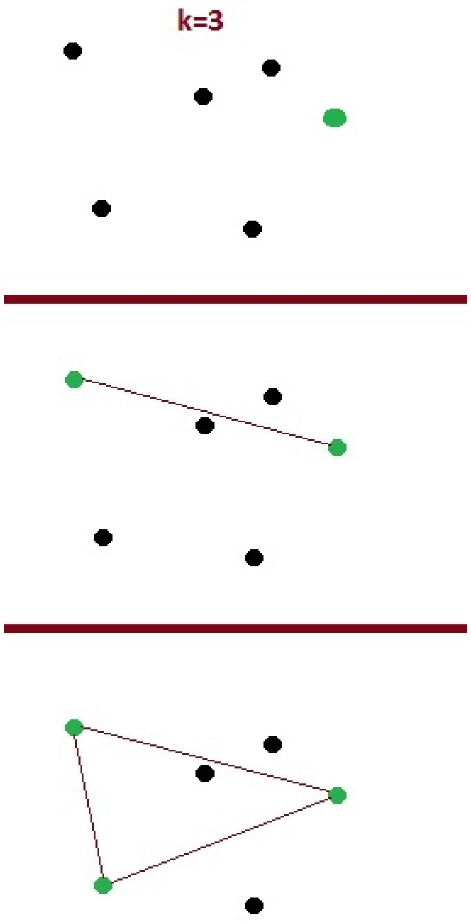
Diversity function		Offline ApproxFactor	Composable Coreset Approx factor [Our Results]
Remote-edge	Minimum Pairwise Distance	$O(1)$ [Tmair 91][White 91] [Ravi et al 94]	$\boldsymbol{O(1)}$
Remote-clique	Sum of Pairwise Distances	$O(1)$ [Hassin et al 97]	$\boldsymbol{O(1)}$
Remote-tree	Weight of MST	$O(1)$ [Halldorsson et al 99]	$\boldsymbol{O(1)}$
Remote-cycle	Weight of minimum TSP	$O(1)$ [Halldorsson et al 99]	$\boldsymbol{O(1)}$
Remote-star	Weight of minimum star	$O(1)$ [Chandra&Halldorsson 01]	$\boldsymbol{O(1)}$
Remote-Pseudoforest	Sum of the distance of each point to its nearest neighbor	$O(\log k)$ [Chandra&Halldorsson 01]	$\boldsymbol{O(\log k)}$
Remote-Matching	Weight of minimum perfect Matching	$O(\log k)$ [Chandra&Halldorsson 01]	$\boldsymbol{O(\log k)}$
Max-Coverage	How well the points cover each coordinate $\sum_{i=1}^{d} \max_{p \in S} p_i$	$O(1)$ [Feige 98]	**No Composable Coreset of Poly size in $\boldsymbol{k}$ with app. factor $\boldsymbol{\frac{\sqrt{k}}{\log k}}$**

Review of Offline Algorithms

- We have a set of n point P
- Goal: find a subset S of size k which maximizes the diversity

The Greedy Algorithm

- Used for minimum-pairwise distance



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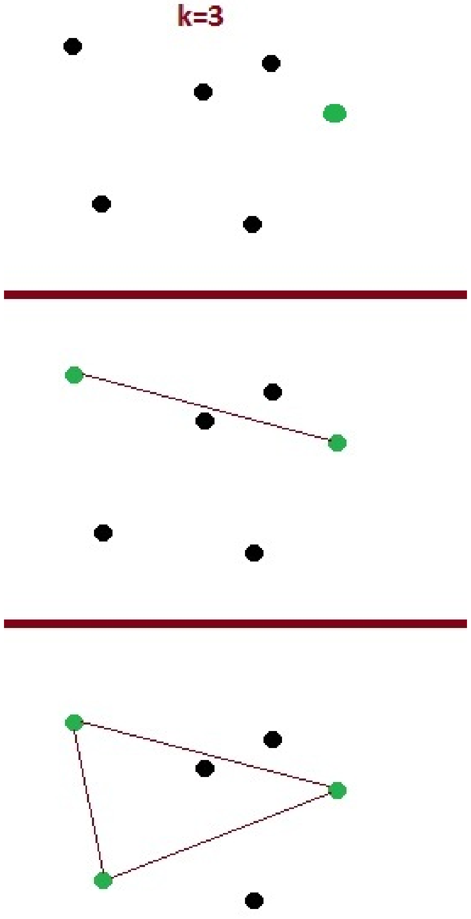
  - Choose an arbitrary point
  - Repeat k-1 times
    - Add the point whose minimum distance to the currently chosen points is maximized



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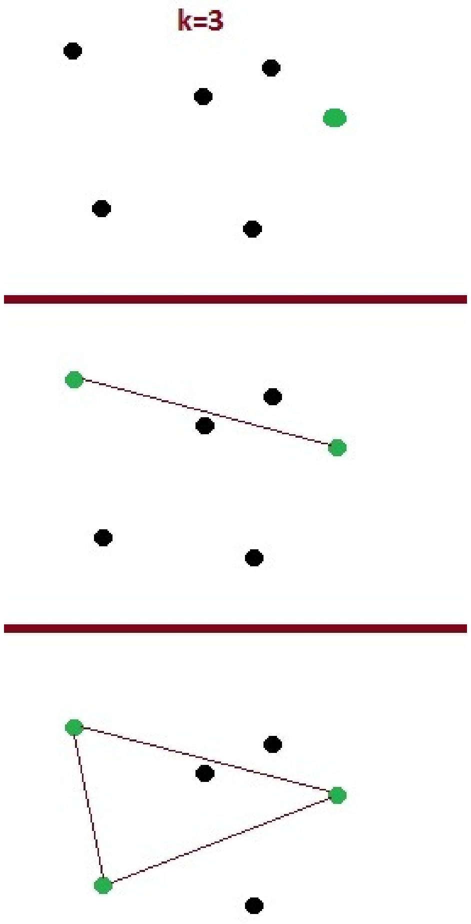
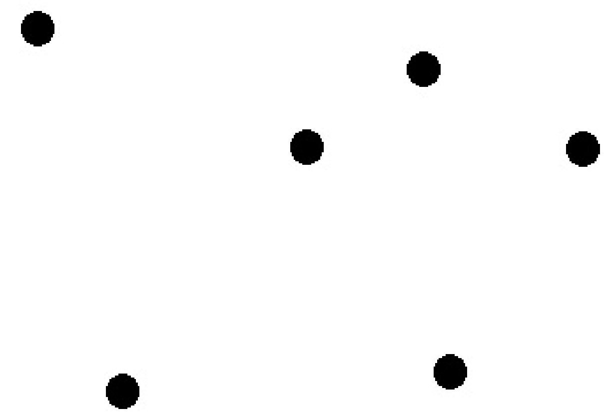
- Remote-edge: computes a 2-approximate set



Local Search Algorithm

- Used for sum of pairwise distances



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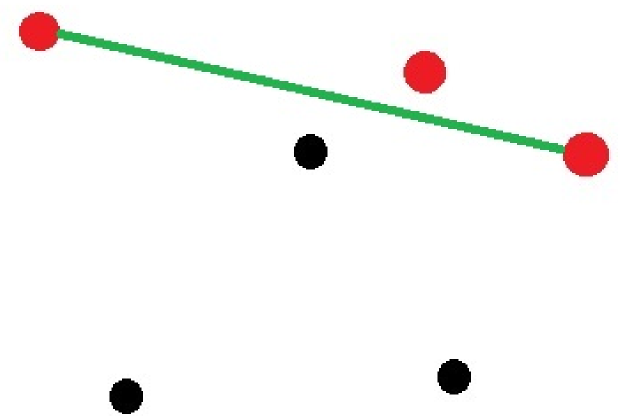
  - Initialize S with an arbitrary set of k points which contains the two farthest points



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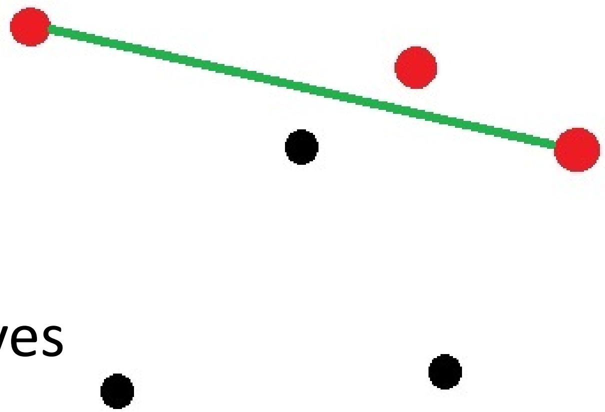
  - While there exists a swap that improves diversity by a factor of $\left(1 + \frac{\epsilon}{n}\right)$



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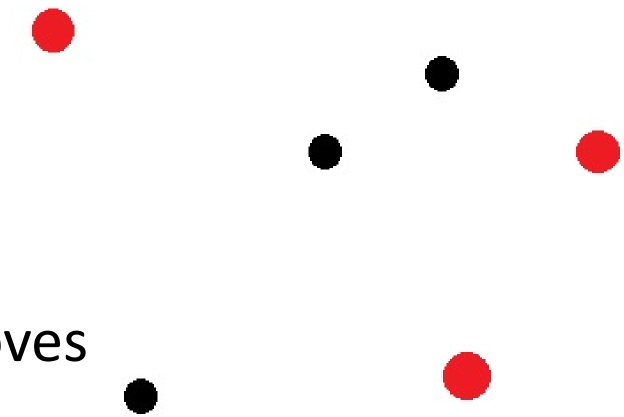

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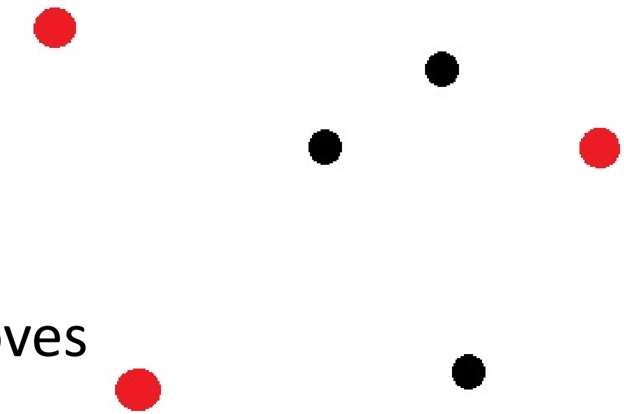

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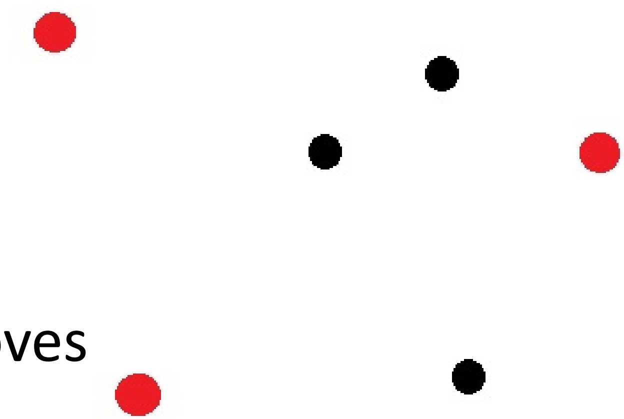


- For Remote-Clique
  - Number of rounds: $\log_{\left\{1+\frac{\epsilon}{n}\right\}} k^2 = O(\frac{n}{\epsilon}\log k)$
  - Approximation factor is constant.

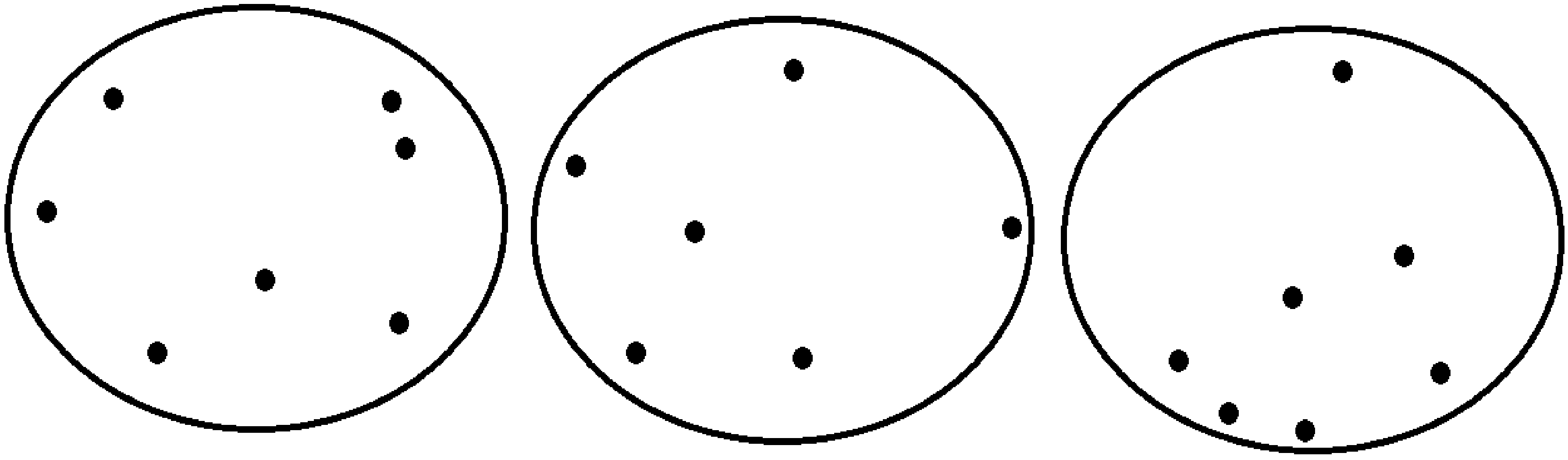
Composable Core-sets

- Greedy Algorithm Computes a 3-composable core-set for minimum pairwise distance

- Local Search Algorithm Computes a constant factor composable core-set for sum of pairwise distances.

Proof Idea

Let $P_1, \cdots, P_m$ be the set of points , $P = \cup P_i$



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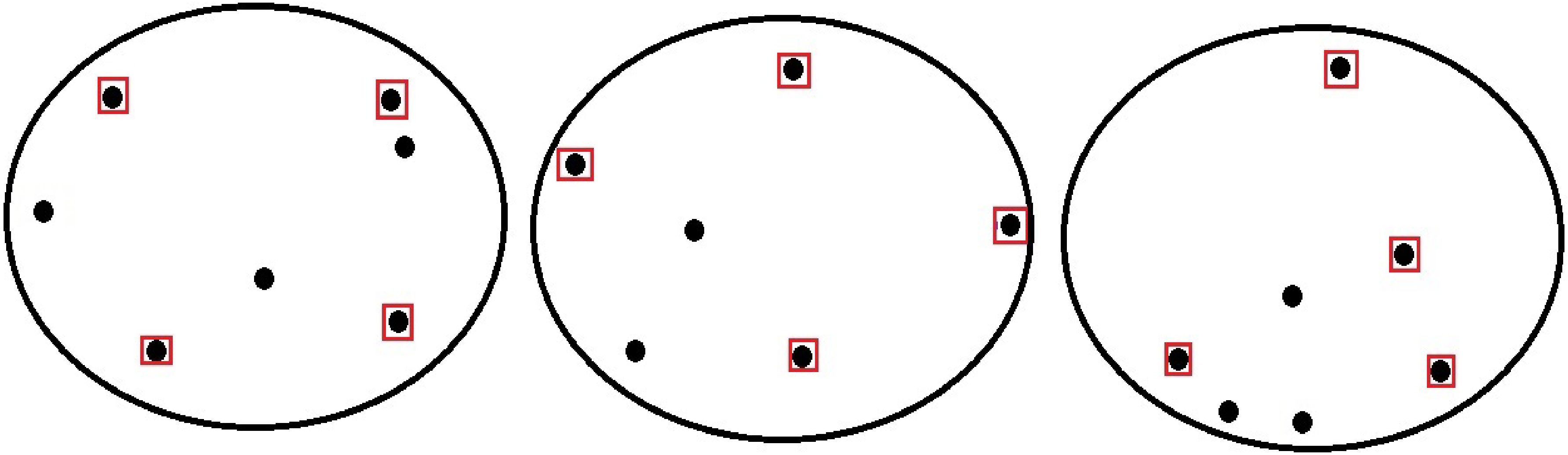
$S_1, \cdots, S_m$ be their core-sets, $S = \bigcup S_i$

Goal: $div_k(S) \geq div_k(P)$ / c



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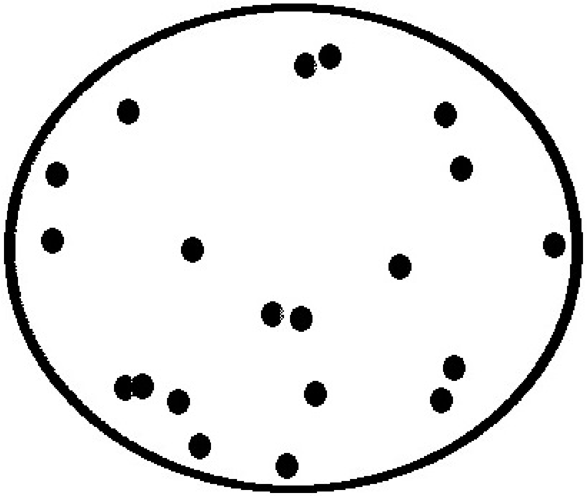

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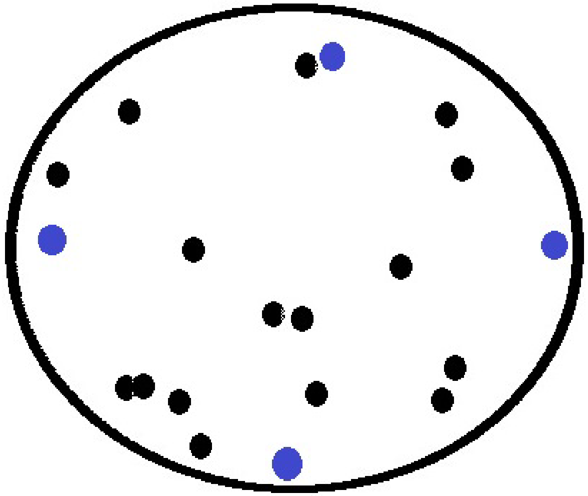

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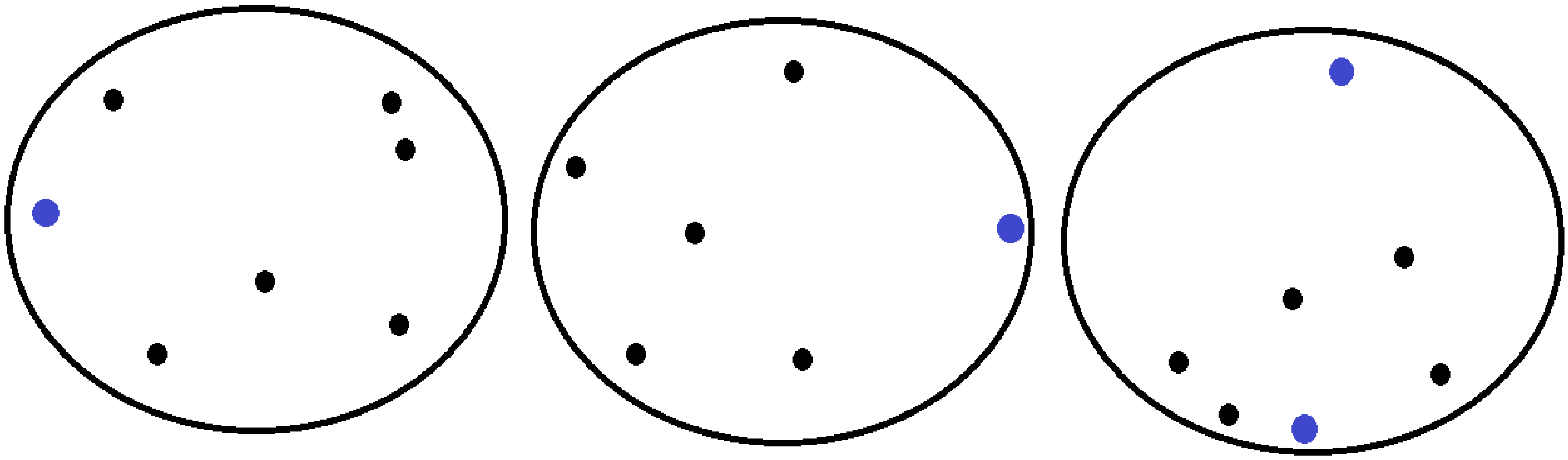

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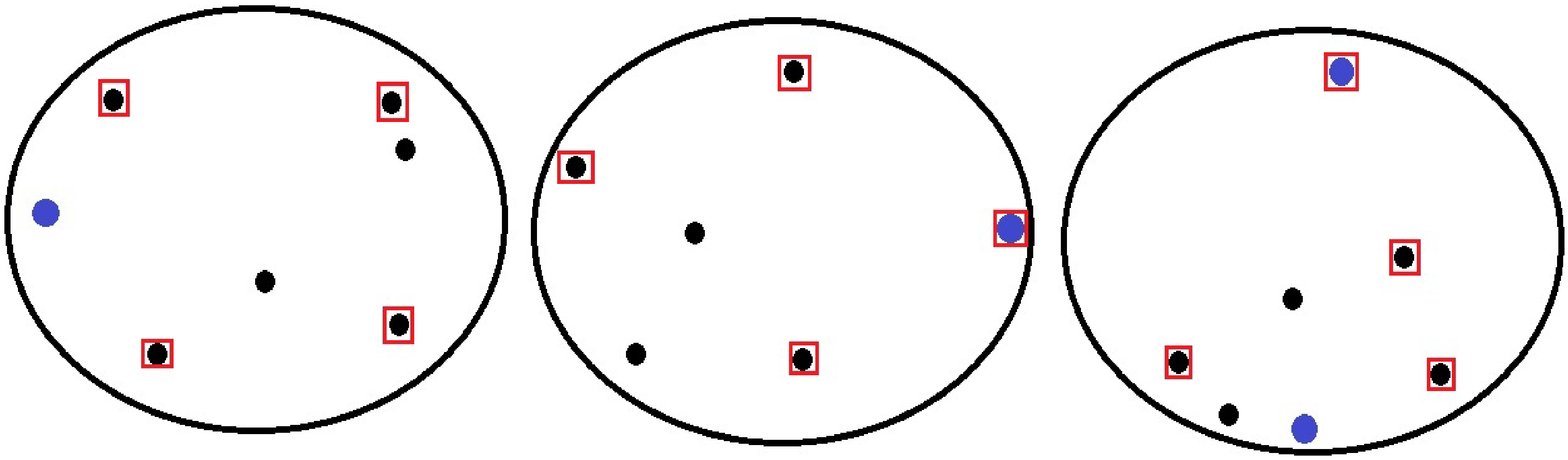
Let r be their maximum diversity , $r = \max_i div(S_i)$,

Goal: $div_k(S) \geq div_k(P)$ / c

Goal: $div_k(S) \geq div(OPT)$ / c

Note: $\mathrm{div_k}(S) \geq r$



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Let $P_1, \cdots, P_m$ be the set of points , $P = \bigcup P_i$

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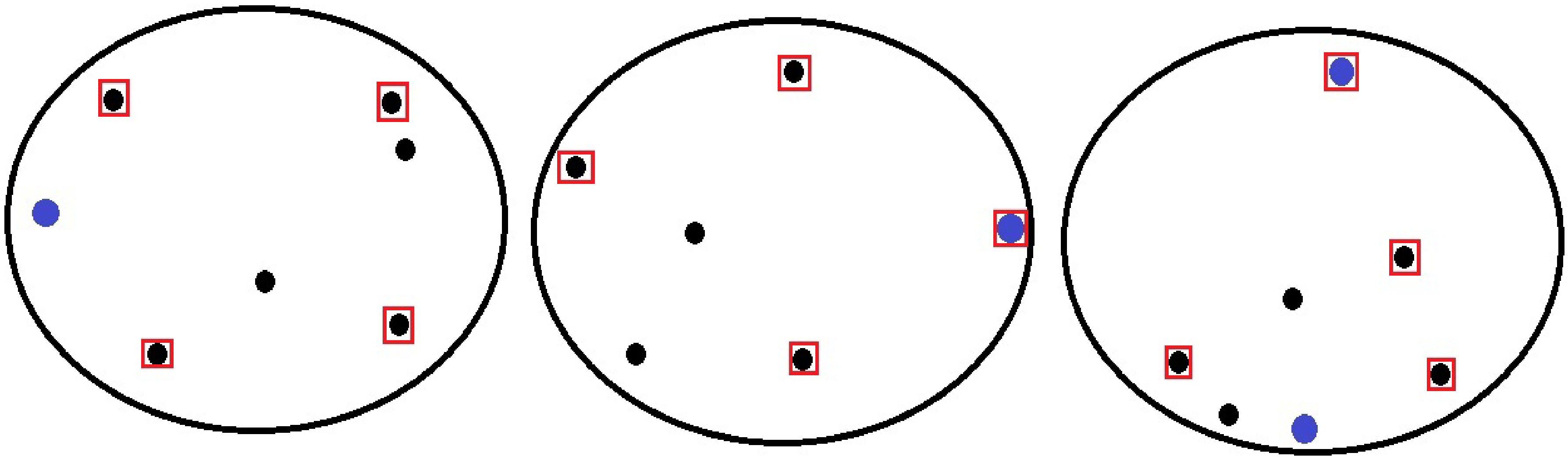

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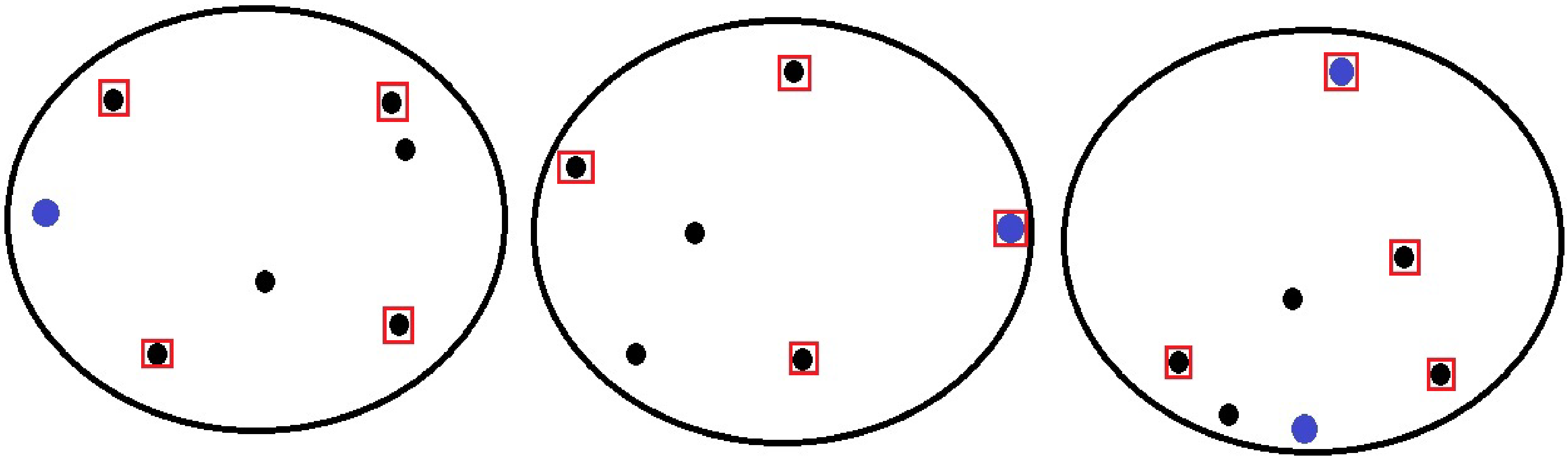

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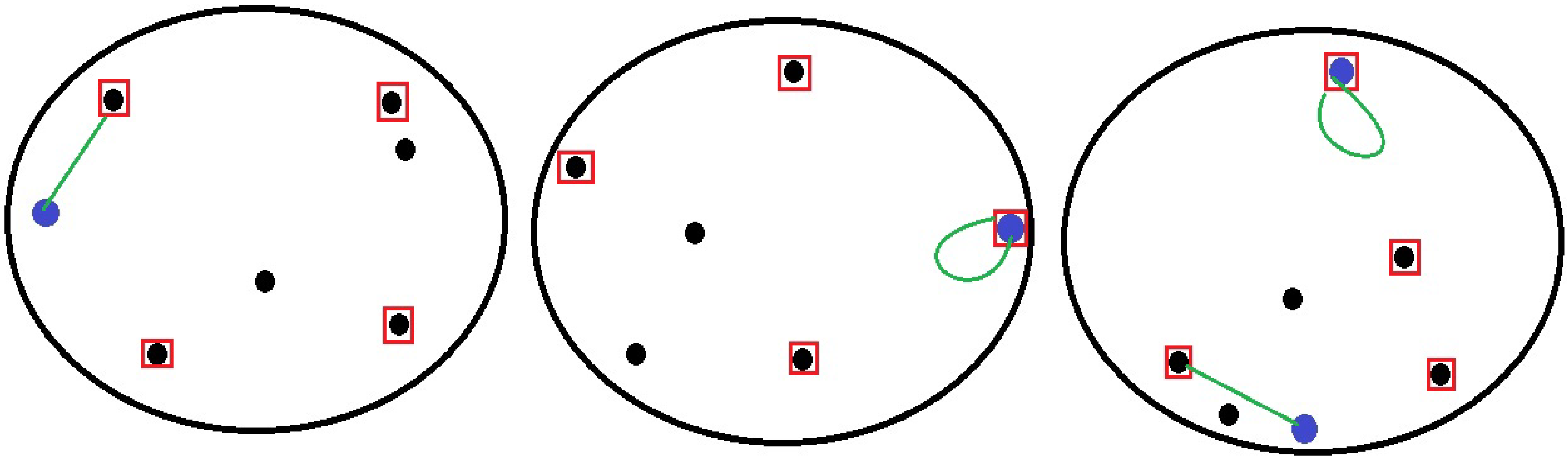

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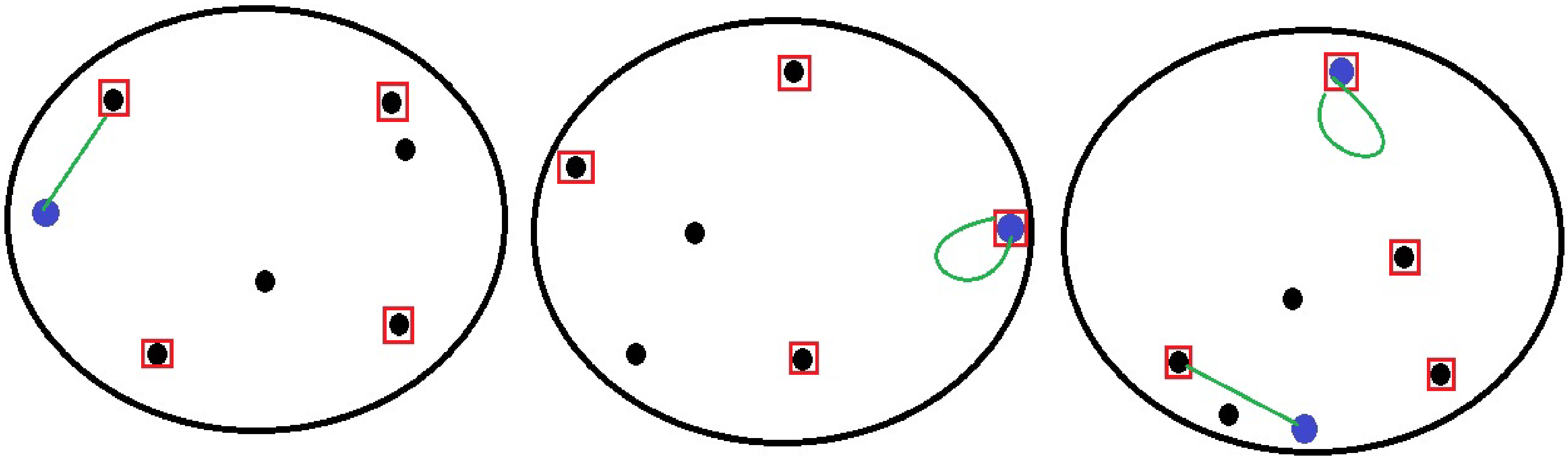

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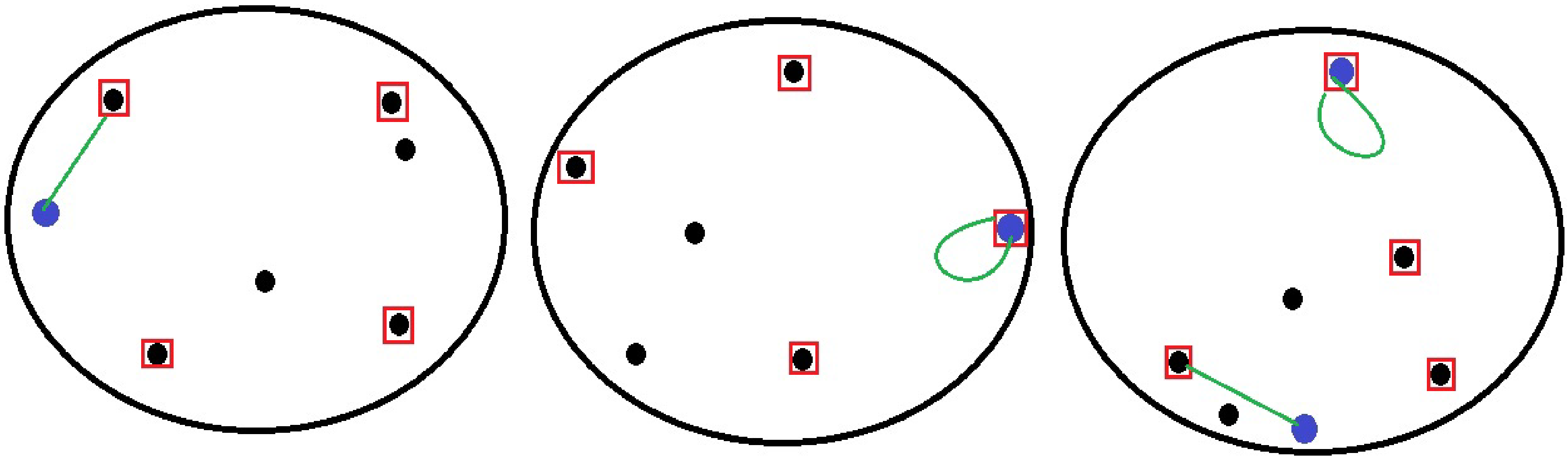

Maximum k-Coverage

- A set of n points P in d-dimensional space
- Each dimension corresponds to a feature.
- Goal: choose a set of k points S in P which maximizes the total coverage:
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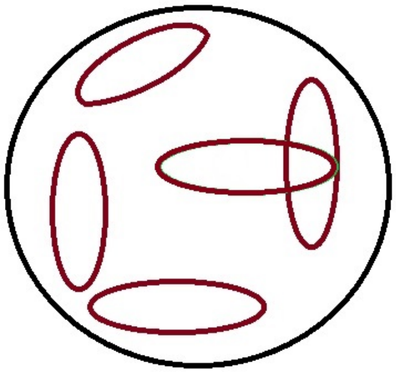

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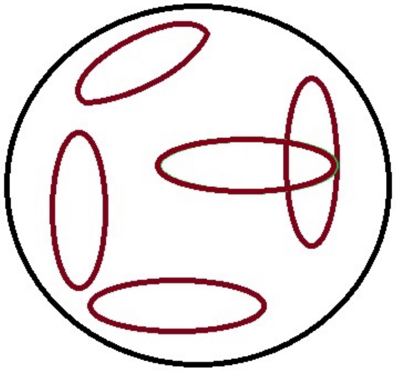

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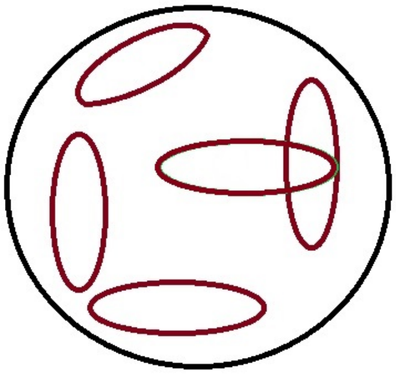

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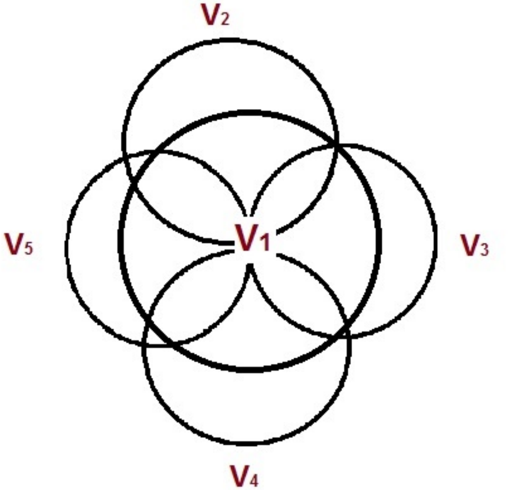

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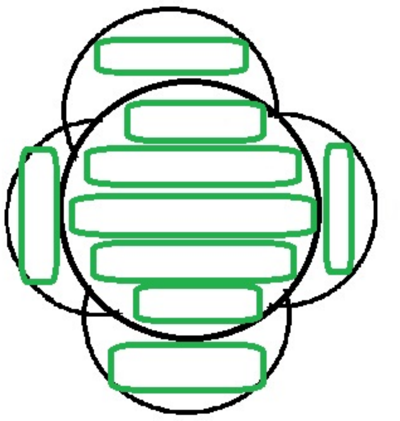

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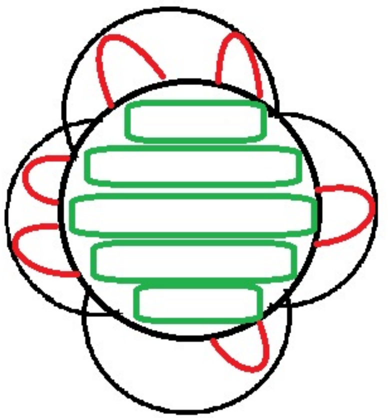

- Using k sets everything in $\cup V_i$ can be covered, that is $O(k^{3/2})$ elements.
- Using core-sets only $|V_1| + k \log k = \mathrm{O}(\mathrm{k} \log \mathrm{k})$ can be covered

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  - Better approximation factors?

Thank You!

Questions?